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Tubular D-branes in Salam-Sezgin model

Chanju Kim

Department of Physics, Ewha Womans University
Seoul 120-750, Korea
E-mail: cjkim@ewha.ac.kr

Yoonbai Kim and O-Kab Kwon

BK21 Physics Research Division and Institute of Basic Science
Sungkyunkwan University, Suwon 440-746, Korea
E-mail: yoonbai@skku.edu, okab@skku.edu

Abstract: We study DBI-type effective theory of an unstable D3-brane in the background manifold $\mathbb{R}^{1,1} \times \mathcal{M}_2$ where $\mathcal{M}_2$ is an arbitrary two-dimensional manifold. We obtain an exact tubular D2-brane solution of arbitrary cross sectional shape by employing $1/\cosh$ tachyon potential. When $\mathcal{M}_2 = \mathrm{S}^2$, the solution is embedded in the background geometry $\mathbb{R}^{1,3} \times \mathrm{S}^2$ of Salam-Sezgin model. This tachyon potential shows a unique property that an array of tachyon soliton solutions has a fixed period which is independent of integration constants of the equations of motion. The thin BPS limit of the configurations leads to supertubes of arbitrary cross sectional shapes.

Keywords: Solitons Monopoles and Instantons, Tachyon Condensation.

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Contents

1. Introduction

When D-branes are wrapped on some nonsupersymmetric cycles in the moduli space of compactified manifolds, e.g., K3 or Calabi-Yau manifolds, they are dissociated and form several stable D-branes among which each is wrapped on a supersymmetric cycle [1, 2]. A representative example is a Dp-brane and one of the directions is compactified as a circle of radius R. At the critical radius $R = \sqrt{2}$, it decays into a pair of D$(p-1)\bar{\mathrm{D}}(p-1)$ branes [3] situated at diametrically opposite points. In terms of boundary conformal field theory (BCFT), this phenomenon is described by a marginal deformation interpolating between the original unstable Dp-brane and the D$(p-1)\bar{\mathrm{D}}(p-1)$ pair [1].

In the context of effective field theory (EFT) where the instability of Dp-brane is represented by condensation of a tachyon field, the Dirac-Born-Infeld (DBI) type effective action [4, 5] with the choice of $1/\cosh$ potential [6]–[8] can reproduce this as an array of static tachyon kink-antikinks with fixed period $2\pi R$ [9, 10]. Only in this EFT, the tachyon profile in BCFT and the tachyon field configuration in EFT has one-to-one correspondence by an explicit point transformation [11] so that the identification between them can clearly be made at the classical level [12]. When the gauge field is turned on on the unstable D-brane, the period of the D$(p-1)\bar{\mathrm{D}}(p-1)$ array starts changing from $2\pi R$ to a larger value and it eventually goes to infinity when the electric field approaches the critical value [10, 12, 13]. For each value of the electric field, the period is fixed and independent of other integration constants of the equation of motion in EFT with $1/\cosh$ potential [10, 13].

On the other hand, when the shape of tachyon potential is chosen to be different from $1/\cosh$, the property of fixed periodicity seems likely to be lost even in pure tachyon case [14]. In this paper, we will show that, in the EFT with DBI type action, the $1/\cosh$ tachyon potential should uniquely be chosen in order to keep this periodic property.

The codimension-one D-branes which are represented as kinks or antikinks in EFT become BPS objects only in zero thickness limit [15] (except the case of the composite of Dp-brane and fundamental string (F1) fluid with critical electric field [10, 13]). This is also true for codimension two or three objects such as vortex-antivortex pairs or monopole-antimonopole pairs though BCFT description may not be explicit [1, 2].

In this paper we will consider another configuration, a generation of a tubular brane on $\mathbb{R}^1 \times S^2$ of which the thin limit is a supertube [16] along the equator of S^2. The tubular D2-brane can have an arbitrary cross sectional shape [17] and it is natural to expect this property to appear also in tachyon tubes from an unstable D3-brane [18]. We will find thick tachyon tube solutions of arbitrary cross section from an unstable D3-brane on the background manifold $\mathbb{R}^{1,1} \times \mathcal{M}_2$, and discuss BPS supertube limit by taking the thickness to be zero.

The case of $\mathcal{M}_2 = S^2$ is of our particular interest. In this case the base spacetime is embedded in $\mathbb{R}^{1,3} \times S^2$ of six-dimensional Salam-Sezgin model [19]. Recently Salam-Sezgin model has been studied in various contexts [20]–[29]. In particular, several investigations have been made in relation with the vacuum structure. A consistent S^2 reduction of the Salam-Sezgin model was performed and its four-dimensional spectrum was analyzed [23]. A new family of supersymmetric vacua in the six-dimensional chiral gauged $N = (1, 0)$ supergravity was discovered, of which the generic form is $AdS_3 \times S^3$, and in this scheme $\mathbb{R}^{1,3} \times S^2$ can be viewed as a fine-tuning [22]. Uniqueness of the Salam-Sezgin vacuum among all nonsingular backgrounds with four-dimensional Poincaré, de Sitter, or anti de Sitter invariance was proved [25].

Our analysis is based on the EFT, and it is unclear whether or not the obtained tube solution can be a consistent BCFT solution in the background of string theory. Since higher-dimensional origin of Salam-Sezgin model has also been obtained [24, 29], this important issue should be addressed in a consistent manner.

The rest of paper is organized as follows. In section 2, we prove uniqueness of the tachyon potential for fixed periodicity of the array of tachyon soliton-antisoliton pairs. In section 3, we obtain exact tachyon tube solutions on $\mathbb{R}^{1,1} \times \mathcal{M}_2$, where $\mathcal{M}_2$ is a two-dimensional manifold, and discuss their BPS limit. In section 4, we consider the case $\mathcal{M}_2 = S^2$ in more detail. We conclude in section 5 with brief discussion.

2. Tachyon potential of D-brane wrapped on a cycle

The effective tachyon action for an unstable Dp-brane system [4, 5] is

$$S = -\mathcal{T}_p \int d^{p+1}x \; V(T) \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T)} \,, \tag{2.1}$$

where $g_{\mu\nu}$ is the metric given from the closed string sector, $T(x)$ is tachyon field, and $F_{\mu\nu}$ field strength tensor of a gauge field A_μ on the Dp-brane, of which the constant piece can also be interpreted as NS-NS two form field. We set $2\pi\alpha' = 1$ and then $\mathcal{T}_p$ is tension of the Dp-brane.

JHEP05(2004)020

Since tachyon potential measures variable tension of the unstable D-brane, it should be a runaway potential connecting

$$V(T=0)=1 \qquad \text{and} \qquad V(T=\infty)=0\,. \tag{2.2}$$

Various forms of it have been proposed, e.g., $V(T)\sim e^{-T^2}$ from boundary string field theory [30] or $V(T)\sim e^{-T}$ for large T in ref. [31]. In this paper, we employ the form [6, 7, 8]

$$V(T)=\frac{1}{\cosh\left(T/R\right)} \tag{2.3}$$

which connects the small and the large T behaviors smoothly. Here, R is $\sqrt{2}$ for the non-BPS D-brane in the superstring and 2 for the bosonic string. This form of the potential has been derived in open string theory by taking into account the fluctuations around $\frac{1}{2}$S-brane configuration with the higher derivatives neglected, i.e., $\partial^2 T=\partial^3 T=\cdots=0$ [11, 32, 33].

Most of the physics of tachyon condensation is irrelevant to the detailed form of the potential once it satisfies the runaway property and the boundary values (2.2). For example, both the basic runaway behavior of rolling tachyon solutions [31] and the BPS nature of tachyon kinks with zero thickness [15] are attained irrespective of the specific shape of the potential which just reflects a detailed decaying dynamics of the unstable D-brane.

On the other hand, there are also some nice features of the form (2.3) in addition to the fact that it is derived from open string theory in a specific regime. Under the 1/cosh tachyon potential (2.3), exact solutions are obtained for rolling tachyon [7, 13] and tachyon kink solutions on unstable Dp with a coupling of abelian gauge field for arbitrary p [9, 10, 14, 13, 18]. Another useful property may be the observation that some of the obtained classical solutions $T(x)$ in the EFT (2.1), e.g., rolling tachyons [9] and tachyon kinks [12], can be directly translated to BCFT tachyon profiles $\tau(x)$ in open string theory described by the following relation obtained in ref. [11],

$$\frac{\tau(x)}{R}=\sinh\left(\frac{T(x)}{R}\right). \tag{2.4}$$

In this section, we would like to discuss another important feature of the $1/\cosh$ potential (2.3), which is not shared by any other form. Among the tachyon soliton solutions in the effective theory, various tachyon array solutions of codimension one have been found, namely, those formed by pure tachyon kink-antikink [3, 1, 9, 10], tachyon kink-antikink coupled to the electromagnetic field [10, 12, 13], and tachyon tube-antitube [18]. An interesting property of all these solutions is that, with the $1/\cosh$ potential in the EFT, the periodicity of the array is independent of any integration constant of the equation of motion, much like the case of simple harmonic oscillator. Here we will show that the converse is also true by adopting the similar line of argument to the case of simple harmonic oscillator: imposing the condition that the periodicity of the tachyon array solutions should be independent of the integration constant of the equation of motion uniquely determines the tachyon potential as eq. (2.3). This property is necessary if one wishes to identify the array solution as a configuration on a circle or a sphere of a fixed radius [12, 34].

JHEP05(2004)020

To begin with, we recall that the relevant equation for all the array solutions with $T = T(x)$ and $F_{\mu\nu}$ is summarized by a single first-order ordinary differential equation

$$\mathcal{E} = \frac{1}{2}T'^2 + \frac{1}{h}U(T)\,, \tag{2.5}$$

where $U = V^2(T)$. (See ref. [10, 13, 18] and also (3.4) in the next section.) For the array of kink-antikink, two parameters $\mathcal{E}$ and h are

$$\mathcal{E} = -\frac{\beta_p}{2\alpha_p}\,, \qquad h = -\frac{2\alpha_p\gamma_p^2}{\mathcal{T}_p^2}\,, \tag{2.6}$$

where $\beta_p = -\det(\eta_{\mu\nu} + F_{\mu\nu})$, α_p is cofactor of 11-component of the matrix $-(\eta + F)_{\mu\nu}$, and γ_p an integration constant [10, 13]. For the array of tube-antitube, $\mathcal{E}$ and h are

$$\mathcal{E} = -\frac{1}{2}\,, \qquad h = -\frac{2\alpha^2\beta^2}{\mathcal{T}_3^2}\,, \tag{2.7}$$

where α is D0 charge density per unit length and β an integration constant [18]. (See also (3.4) in the next section.) Then, for our purpose, the coefficient h in front of the potential is negative and to be varied, and $\mathcal{E}$ is regarded as a constant with $1/h < \mathcal{E} < 0$.

Let us require the period to be independent of h in eq. (2.5). Denoting the period as ζ, we have

$$\begin{aligned}\frac{\pi}{2}\zeta &= \int_0^{T_{\max}} \frac{dT}{\sqrt{2[\mathcal{E} - U(T)/h]}} \\ &= \int_{U_0}^{h\mathcal{E}} \frac{dT/dU}{\sqrt{2(\mathcal{E} - U/h)}} dU\,,\end{aligned} \tag{2.8}$$

where $T_{\max}$ is the maximum value of the tachyon field, $\mathrm{U}(T_{\max}) = h\mathcal{E}$, and $U_0 = \mathrm{U}(T = 0)$. It turns out to be convenient to define the variable $\eta = h\mathcal{E}$. Then eq. (2.8) becomes

$$\frac{\pi}{2}\zeta = \frac{1}{\sqrt{2|\mathcal{E}|}} \int_{U_0}^{\eta} \frac{\sqrt{-\eta}}{\sqrt{\eta - U}} \frac{dT}{dU} dU\,. \tag{2.9}$$

If both sides of this equation are divided by $\sqrt{(-\eta)(U - \eta)}$ and integrated with respect to η from U_0 to U,

$$\begin{aligned}\frac{\pi\zeta}{2}\int_{U_0}^{U} \frac{d\eta}{\sqrt{\eta^2 - U\eta}} &= \frac{1}{\sqrt{2|\mathcal{E}|}} \int_{U_0}^{U}\int_{U_0}^{\eta} d\eta dU' \frac{dT(U')/dU'}{\sqrt{(U-\eta)(\eta - U')}} \\ &= \frac{1}{\sqrt{2|\mathcal{E}|}} \int_{U_0}^{U} dU' \int_{U'}^{U} d\eta \frac{dT(U')/dU'}{\sqrt{(U-\eta)(\eta - U')}}\,,\end{aligned} \tag{2.10}$$

where we changed the order of integration in the second line.

It is now elementary to perform the integral (2.10) in the both sides. The result is

$$\pi\zeta \,\mathrm{arccosh}\left(\sqrt{\frac{U_0}{U}}\right) = \frac{\pi T}{\sqrt{2|\mathcal{E}|}}\,, \tag{2.11}$$

i.e.,

$$U(T) = \frac{U_0}{\cosh^2(T/R)}\,, \tag{2.12}$$

where $R = \zeta\sqrt{2|\mathcal{E}|}$. Comparing eq. (2.11) with eq. (2.5), we see that $V(T) = 1/\cosh(T/R)$ as asserted. This property can also be seen clearly after a point transformation (2.4) to the equation (2.5), which, under the specific tachyon potential (2.3), results in

$$\mathcal{E}' = \frac{1}{2}\tau'^2 + \frac{1}{2}\omega^2\tau^2\,, \tag{2.13}$$

where $0 < \mathcal{E}' = -1/h + \mathcal{E}$ and $0 < \omega^2 = -2\mathcal{E}/R^2$. Since both $\mathcal{E}$ and R are fixed but h is a variable, $\mathcal{E}'$ is a positive variable and ω^2 a constant. Therefore, eq. (2.13) is formally equivalent to the expression of the mechanical energy $\mathcal{E}'$ of a 1-dimensional simple harmonic oscillator with unit mass of which the position is τ at time x. According to the proof in ref. [35], its period $2\pi/\omega$ is independent of the value of $\mathcal{E}'$ only for the simple harmonic oscillator.

This periodic property of the array configurations in the effective field theory is desirable if we wish to identify the array solution as a pair of $\mathrm{D}(p-1)\bar{\mathrm{D}}(p-1)$ obtained from an unstable Dp-brane wrapped on a cycle in the context of string theory [1, 2, 12]. (Note also from eq. (2.12) that the compactified length ζ varies as the electromagnetic field changes.) In this sense, our proof in this section tells the uniqueness of the $1/\cosh$ tachyon potential (2.3) for the tachyon field in eq. (2.1) in studying the generation of codimension one extended objects on nonsupersymmetric cycles. In section 3, we will find a family of tachyon tube solutions with such periodicity on $\mathbb{R}^1 \times \mathcal{M}_2$, and in section 4, will demonstrate that single tachyon tube on $\mathbb{R}^1 \times \mathrm{S}^2$ forms a thin tubular object of which the geometry is $\mathbb{R}^1 \times \mathrm{S}^1$ in the BPS limit.

3. Tachyon tubes of arbitrary cross section and BPS limit

In this section we consider tachyon tube configurations in the theory described by the DBI type action (2.1) on $\mathbb{R}^{1,1} \times \mathcal{M}_2$ in the coordinate system (t, z, u, v) with metric

$$ds^2 = -dt^2 + dz^2 + du^2 + f(u)^2 dv^2\,, \tag{3.1}$$

where $f(u)$ is an arbitrary function. Depending on $f(u)$ the two-dimensional manifold $\mathcal{M}_2$ defined by (u, v)-coordinates can be either compact or noncompact. In flat $\mathbb{R}^2$ case ($f(u) = u^2$) tachyon tube solutions were obtained in ref. [18]. Here we will show that there exist various tachyon tubes with arbitrary cross sectional shapes, as in the case of supertubes [17]. In particular, configurations on S^2 will be considered in greater detail in the subsequent section.

As an ansatz, we assume that the fields are dependent only on the coordinate u and $F_{0u} = F_{uv} = F_{zu} = 0$. Then nonvanishing fields are $T = T(u)$, $F_{0z} \equiv E_z(u)$, $F_{0v} \equiv E_v(u)$, and $F_{vz}/f \equiv B_u(u)$. With the ansatz, the Bianchi identity dictates E_z and E_v to be constants, and $B_u \sim 1/f$. In this paper we further restrict our interest to looking for the configurations with the critical value for E_z and vanishing E_v[1] so that we have

$$|E_z| = 1\,, \qquad E_v = 0\,, \qquad B_u = \frac{\alpha}{f}\,, \tag{3.2}$$

where α is an arbitrary D0 charge density at $f = 0$ and due to that the Bianchi identity $\nabla \cdot \mathbf{B} = 0$ fails at $f = 0$.

Substituting eq. (3.2) with $T = T(u)$ into the action (2.1), we find that the action is independent of the metric function $f(u)$,

$$S = -\mathcal{T}_3 \alpha \int dtdzdv \int du\, V(T)\sqrt{1 + T'^2}\,, \tag{3.3}$$

where the prime denotes differentiation with respect to the variable u. Then the equation of motion reduces to

$$-\, fT_{uu} \equiv \mathcal{T}_3 \frac{V\alpha}{\sqrt{1+T'^2}} = \beta\alpha^2\,, \tag{3.4}$$

where β is a nonnegative constant and T_{uu} the uu-component of pressure.

For the solutions of eq. (3.4), many components of energy-momentum tensor $T_{\mu\nu}$ and conjugate momenta of the gauge field Π_i vanish,

$$T_{0z} = T_{0u} = T_{zu} = T_{zv} = T_{uv} = T_{vv} = \Pi_u = \Pi_v = 0\,. \tag{3.5}$$

The nonvanishing components share the same functional T-dependence (and hence the same u-dependence) except T_{uu} in eq. (3.4),

$$\begin{aligned} fT_{00} &= (f^2 + \alpha^2)\Sigma(u)\,, \\ T_{0v} &= \alpha f \Sigma(u)\,, \\ T_{zz} &= -f\Sigma(u)\,, \\ \Pi &= f^2 \Sigma(u)\,, \end{aligned} \tag{3.6}$$

where $\Pi \equiv \Pi_z$ and

$$\Sigma(u) = \beta(1 + T'^2) = \frac{1}{\beta\alpha^2}(\mathcal{T}_3 V)^2\,. \tag{3.7}$$

The energy per unit length then satisfies the relation

$$\mathcal{E} = \int dudv\, fT_{00} = Q_{\rm F1} + \int dudv\, \Pi B^2\,, \tag{3.8}$$

where $Q_{\rm F1}$ is F1 charge per unit length,

$$Q_{\rm F1} = \int dudv\, \Pi\,. \tag{3.9}$$

Note that eq. (3.8) holds irrespective of the form of both the tachyon potential $V(T)$ and the metric function $f(u)$. We will shortly see that, with the $1/\cosh$-type potential (2.3) (and only with this potential), the second term of eq. (3.8) is identified as D0 charge per unit length.

[1] Nonvanishing E_v ($E_v^2 < \alpha^2$) can easily be understood through a boost transformation along z-direction and the corresponding object is a helical tachyon tube [18].

Now let us discuss the solution of eq. (3.4) in detail. With the form of tachyon potential (2.3), it is easy to obtain the exact solution

$$\sinh\left(\frac{T(u)}{R}\right) = \pm\left[\sqrt{\left(\frac{\mathcal{T}_3}{\alpha\beta}\right)^2 - 1}\,\cos\left(\frac{u}{R}\right)\right], \tag{3.10}$$

where we imposed the condition $T'(0) = 0$ for regularity. This solution represents a coaxial array of tubular kink-antikink with periodicity $2\pi R$. Note that the period is independent of integration constants α and β. This is consistent with the discussion on the unique property of 1/cosh tachyon potential in section 2.

For the solution, the quantity $\Sigma(u)$ of eq. (3.7) is given by

$$\Sigma(u) = \beta\frac{(\mathcal{T}_3/\alpha\beta)^2}{1+[(\mathcal{T}_3/\alpha\beta)^2-1]\cos^2(u/R)}\,. \tag{3.11}$$

The energy (tube tension) of a single kink (per unit length) is then calculated as

$$\begin{aligned}\mathcal{E}_2^{(n)} &= \int dv\int_{(n-1)\pi R}^{n\pi R} du\, fT_{00}\\ &= \beta\int dv\int_{(n-1)\pi R}^{n\pi R} du\frac{(\mathcal{T}_3/\alpha\beta)^2(f^2+\alpha^2)}{1+[(\mathcal{T}_3/\alpha\beta)^2-1]\cos^2(u/R)}\,,\end{aligned} \tag{3.12}$$

and the string charge per unit length is

$$\begin{aligned}Q_{\rm F1}^{(n)} &= \int dv\int_{(n-1)\pi R}^{n\pi R} du\,\Pi\\ &= \beta\int dv\int_{(n-1)\pi R}^{n\pi R} du\,\frac{(\mathcal{T}_3/\alpha\beta)^2\, f^2}{1+[(\mathcal{T}_3/\alpha\beta)^2-1]\cos^2(u/R)}\,.\end{aligned} \tag{3.13}$$

Though the energy and the string charge are not calculable explicitly for general $f(u)$, the difference $\mathcal{E}_2^{(n)} - Q_{\rm F1}^{(n)}$ is quite simple and can be calculated explicitly,

$$\begin{aligned}\mathcal{E}_2^{(n)} - Q_{\rm F1}^{(n)} &= \beta\alpha^2\int dv\int_{(n-1)\pi R}^{n\pi R} du\,\frac{(\mathcal{T}_3/\alpha\beta)^2}{1+[(\mathcal{T}_3/\alpha\beta)^2-1]\cos^2(u/R)}\\ &= \pi\alpha R\mathcal{T}_3\int dv\\ &\equiv Q_{\rm D0}^{(n)}\,,\end{aligned} \tag{3.14}$$

which coincides with the D0-brane charge per unit length. Note that it is independent of $f(u)$ or β. Therefore we have a BPS-like sum rule

$$\mathcal{E}_2^{(n)} = Q_{\rm F1}^{(n)} + Q_{\rm D0}^{(n)}\,. \tag{3.15}$$

In addition, each unit tube (or antitube) carries angular momentum per unit length

$$L^{(n)} = -\alpha\beta\int dv\int_{(n-1)\pi R}^{n\pi R} du\frac{(\mathcal{T}_3/\alpha\beta)^2\, f^2}{1+[(\mathcal{T}_3/\alpha\beta)^2-1]\cos^2(u/R)}\,, \tag{3.16}$$

which is proportional to the string charge, i.e., $L^{(n)} = -\alpha Q_{\rm F1}^{(n)}$.

It is well-known that the supertube solution of cylindrical symmetry is a BPS object preserving 1/4-supersymmetry [16] and this BPS nature is not disturbed for tubular branes with arbitrary cross sectional shape [17]. In the above, we obtained the tachyon tube solution for which the u-coordinate dependence is arbitrary. Since it is given by the configuration of coaxial array of tube-antitubes with nonzero thickness (3.10), it may not be a BPS object despite of the BPS like sum rule (3.15). To see whether the configuration is a BPS object, we look into the stress components on (u, v)-plane. From eqs. (3.5) and (3.4), we find that T_{uv} and T_{vv} vanish but T_{uu} does not. If we accept vanishing of all stress components on (u, v)-plane as a strict saturation of the BPS bound of these spinning tachyon tubes, it can be achieved in the limit either $\alpha \to 0$ or $\beta \to 0$. The former is a trivial limit of a fundamental string without D0's and is of no interest, while the latter corresponds to the zero thickness limit of the tachyon tube which becomes the supertube for $\mathcal{M}$ =S^2. Among the other components which are not in the (u, v)-plane, T_{zu} and T_{zv} vanish before taking the zero-thickness BPS limit as in eq. (3.5). On the other hand, the nonvanishing ones in eq. (3.6) become delta functions since

$$\Sigma(u) \xrightarrow{\beta \to 0} \frac{\pi R \mathcal{T}_3}{\alpha} \sum_n \delta\left(u - \left(n - \frac{1}{2}\right)\pi R\right). \tag{3.17}$$

4. Tachyon tubes in the background of Salam-Sezgin vacuum

Here we study the case $\mathcal{M}_2$ =S^2 in more detail. Spheres appear to be a possible candidate for internal space and well-known examples involving S^2 include $\mathrm{AdS}_2 \times \mathrm{S}^2$ and compactifications on some Calabi-Yau manifolds with S^2 as a submanifold. For simplicity, we assume that the other directions are flat, so the background geometry of our interest is $\mathbb{R}^{1,1} \times \mathrm{S}^2$. A representative example relevant with this flat space is $N = 2$ Einstein-Maxwell supergravity in six-dimensional space $\mathbb{R}^{1,3} \times \mathrm{S}^2$ which is known as Salam-Sezgin model [19]. Its low energy limit admits four-dimensional $N = 1$ supergravity which includes chiral fermions and of which the gauge symmetry is $\mathrm{SO}(3) \times \mathrm{U}(1)$. The geometry $\mathbb{R}^{1,3} \times \mathrm{S}^2$ of the Salam-Sezgin vacuum is expressed by

$$ds_6^2 = -dt^2 + dx^2 + dy^2 + dz^2 + \frac{1}{8g^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{4.1}$$

where $0 \le \theta \le \pi$ and $0 \le \varphi \le 2\pi$. There is a constant magnetic field $-1/2g$ on the two sphere inversely proportional to the gauge coupling g of the Salam-Sezgin model. This $N = 2$ supergravity on $\mathbb{R}^{1,3} \times \mathrm{S}^2$ and its variants have recently attracted attention in relation with various topics [20]–[29].

Motivated by the above, we consider a tachyon tube-antitube solution (3.10) on $\mathbb{R}^{1,1} \times \mathrm{S}^2$ described by the coordinates (t, z, θ, φ) embedded in the Salam-Sezgin vacuum, $\mathbb{R}^{1,3} \times \mathrm{S}^2$ (4.1). Since the period of the solution is $2\pi R$ from eq. (3.10), we identify the coordinates in the background metric (3.1) as

$$u = R\theta, \quad v = R\varphi\,, \qquad f = \sin\theta = \sin\left(\frac{u}{R}\right), \tag{4.2}$$

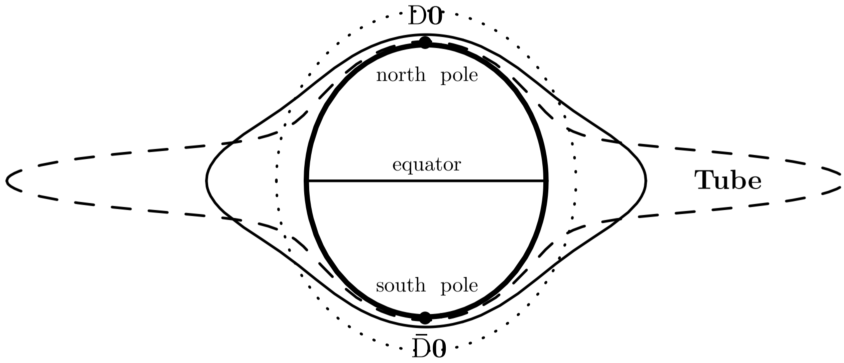

Figure 1: Plots of $\Pi(u)$ and $\mathcal{E}(u) - \Pi(u)$ for $\alpha = 1$ on S^2: dashed line for $\alpha\beta/\mathcal{T}_3 = 0.1$, solid line for $\alpha\beta/\mathcal{T}_3 = 0.3$, and dotted line for $\alpha\beta/\mathcal{T}_3 = 1$. The profiles along the equator represent the tubes and two peaks on both the north and south poles D0 and $\bar{\rm D}$0.

with $0 \le u \le \pi R$ and $0 \le v \le 2\pi R$. Then, g is identified as $g = 1/(2\sqrt{2}R)$, and the resultant background metric becomes eq. (3.1). If the radius R introduced through the tachyon potential (2.3) has a string origin like $R = \sqrt{2}$ or $R = 2$, the gauge coupling g is of the string scale.

From the obtained tachyon profile (3.10), we read that a single tachyon tube lies along the equator ($u = \pi R/2$) and thereby F1 charge density is accumulated there (See figure 1). Linear D0's along z-axis are located at the north pole ($u = 0$) and $\bar{\rm D}$0's at the south pole ($u = \pi R$). An intriguing point is that the energy and the F1 charge per unit length are obtained in closed forms

$$\begin{aligned} \mathcal{E}_2 &= \beta \int_0^{2\pi R} dv \int_0^{\pi R} du \frac{(\mathcal{T}_3/\alpha\beta)^2 \left[\sin^2(u/R) + \alpha^2\right]}{1 + [(\mathcal{T}_3/\alpha\beta)^2 - 1]\cos^2(u/R)} \\ &= 2\pi R \left[\alpha + \frac{1}{\alpha}\frac{1}{1 + (\alpha\beta/\mathcal{T}_3)}\right] \times \pi R \mathcal{T}_3 \,, \end{aligned} \tag{4.3}$$

$$\begin{aligned} Q_{\rm F1} &= \beta \int_0^{2\pi R} dv \int_0^{\pi R} du \frac{(\mathcal{T}_3/\alpha\beta)^2 \sin^2(u/R)}{1 + [(\mathcal{T}_3/\alpha\beta)^2 - 1]\cos^2(u/R)} \\ &= \frac{2\pi R}{\alpha}\frac{1}{1 + (\alpha\beta/\mathcal{T}_3)} \times \pi R \mathcal{T}_3 \,. \end{aligned} \tag{4.4}$$

In the thin limit ($\alpha\beta/\mathcal{T}_3 \to 0$) of a single tachyon tube on S^2 of radius R, F1 charge density is concentrated along the equator like the ring of the Saturn (see the solid and dashed lines in Figure 1). In the opposite limit ($\alpha\beta/\mathcal{T}_3 \to 1$) with $T(u) = 0$ at everywhere, $\mathcal{E}(u) - \Pi(u)$ is evenly distributed (see the dotted line in figure 1). Locations of two point-like peaks due to D0 and $\bar{\rm D}$0 are also indicated at both the north and the south poles, respectively in figure 1.

Product of two-dimensional flat directions R^2 to $\mathbb{R}^{1,1} \times S^2$ is automatic so that the obtained tube solution on $\mathbb{R}^{1,1} \times S^2$ is a tachyon tube solution in Salam-Sezgin model of $\mathbb{R}^{1,3} \times S^2$. Therefore, it describes either a formation of tubular D2-brane from an unstable D3-brane wrapped on $\mathbb{R}^1 \times S^2$ or that of tubular D4-brane from the space-filling unstable D5-brane in Salam-Sezgin model.

5. Conclusion

In this paper we studied DBI-type effective theory of unstable D3-branes and obtained an exact tubular D2-brane solution of arbitrary cross sectional shape. The background manifold of the solution is $\mathbb{R}^{1,1} \times \mathcal{M}_2$ where $\mathcal{M}_2$ is an arbitrary two-dimensional manifold. As the tachyon potential, we employed $1/\cosh$ potential. It was shown that it has a unique property that an array of tachyon soliton solutions has a fixed period which is independent of integration constants of the equations of motion. It also allows us to obtain closed form of solutions. The thin BPS limit of the configurations leads to supertubes of arbitrary cross sectional shapes. In particular, we investigated the case $\mathcal{M}_2 = \mathrm{S}^2$ in more detail for which the solution is embedded in the background geometry $\mathrm{R}^{1,3} \times \mathrm{S}^2$ of Salam-Sezgin model. Since a lifting of this model to ten-dimensional type I supergravity is made, of which weak string coupling limit coincides with an exact string theory solution, the near-horizon geometry of a Neveu-Schwarz (NS) five-brane [29], it would be intriguing to find 9-dimensional analogue of our solution in this 10-dimensional background.

Though our discussions were only about static objects, dynamical generation of $\mathrm{D}(p-1)\bar{\mathrm{D}}(p-1)$ or tubular solution should be achieved as inhomogeneous time-dependent solutions [34]. Until now, it seems incomplete since the solution seems to hit a singularity after time evolution for a finite time [36].

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JHEP05(2004)020